Executive Summary

UGC-Minor Research Project

A Study on Finite State Automata whose Transition Semigroup is Idempotent Generated

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An automaton is a system where energy, material and information are transformed, transmitted and used for performing some functions without direct participation of man. In computer science automaton means 'discrete automaton'and its characteristics are input, output, states, state relation and output relation. Formerly, a finite state automaton is a 5-tuple $\mathcal{A} = (Q, A, \delta, I, F)$, where Q is a finite set called the set of states, A is an alphabet, δ is a function from $Q \times A \to Q$, $I \subseteq Q$ is called the set of initial states and $F \subseteq Q$ is called the set of final states. Let $A*$ denote the free monoid over$A$ and the elements of $A*$ is the set of finite sequence of letters called words. A language is a subset of a free monoid $A*$. The language recognised or accepted by a finite state automaton $\mathcal{A}$ is the set of all words accepted by $\mathcal{A}$ and is denoted by $L(\mathcal{A})$. A formal language (or language) is an abstraction of the general characteristics of a programming language. A language $L \subseteq A*$ is recognisable if it is recognised by a finite state automaton, that is, $L = L(\mathcal{A})$ and these languages are in the lower level of the Chomsky hierarchy. The most celebrated Kleene theorem is considered as the foundation of the theory and it shows that a language is recognizable if and only if it is rational. In the mid of 1960's Schutzemberger established an equivalance between finite state automata and finite semigroups. By this he characterised the star free languages. In the early seventies several authors characterised different type of languages by the semigroup approach. For instance, in the early seventies I. Simon characterised the languages whose semigroup structure is $\mathbb{J}$-trivial and Brzozowski-Simon characterised another class of languages of dot-depth one and locally testable languages. These successes lead to the formation of the famous Eilenberg theorem, established a correspondence between certain families of languages and certain classes of finite semigroups. So the theory of finite semigroups is quite relevant for the development in the theory of finite automata.

A semigroup S is a set with a binary operation which is associative. An element e of S is an idempotent if $e^2 = e$ and a semigroup is idempotent if all of its elements

are idempotents and the idempotents play a key role in analysing the structure of semigroups. Therefore, to determine the structure of a semigroup we must first determine the structure of its idempotents or in other words its structure is encoded in the properties of their idempotents mainly by using Green's relations. For example in the case of inverse semigroups the idempotents commute. Green's relations are a set of equivalence relations defined on semigroups and in groups they are the universal relation. Also a semigroup is idempotent generated if it is generated by the set of all its idempotents and the idempotent generated semigroups have a significant structure.

A detailed literature survey based on the concepts and methods in semigroup theory in the study of finite state automata was made and books and articles related to our work were collected. For instance, the classification of semigroups by various finite state automata over finite number of alphabets and their algebraic characterizations were identified. We have started with two state deterministic finite state automata over two letter alphabet and analysed all the three state deterministic finite state automata over two letter alphabet and their respective transition semigroups. We see that there are ony the five non-isomorphic deterministic two state finite automata $\mathcal{A} = (Q, A, \delta)$ over the alphabet $A = \{a, b\}$ without initial and terminal states, whose transition semigroup is idempotent generated. Moreover we see that transition semigroup of such automata is always a band. Also noted that a two state deterministic finite automata over two letter alphabet such that its transition semigroup is $Tr(\mathcal{A})$ is idempotent generated, then $\mathcal{A}$ is a synchronizing automaton. Through the analysis of three state deterministic finite automata over two letter alphabet we see that there exist 34 non isomorphic idempotent generated semigroup generated by three state deterministic finite automata.

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