Syllabus for the First Degree Programme in Mathematics of the University of Kerala

Semester VI
Graph Theory

CODE: MM 1644 — Instructional hours per week: 3
No. of credits: 3

Overview of the Course: The course has been designed to build an awareness of some of the fundamental concepts in Graph Theory and to develop better understanding of the subject so as to use these ideas skillfully in solving real world problems.

Module 1 A brief history of Graph Theory: The Königsberg bridge problem, the history of the Four Colour Theorem for maps, Contributions to Graph Theory by Euler, Kirchoff, Cayley, Mobius, De Morgan, Hamilton, Erdös, Tutte, Harary, etc. (A maximum of three hours may be allotted to this sub-module. In addition to sections 1.2 and 1.6 of the text, materials for this part can be had from other sources including the internet.)

Graphs: Definition of graph, vertex, edge, incidence, adjacency, loops, parallel edges, simple graph. Representation of graphs, diagrammatic representation, matrix representation (adjacency* matrix and incidence matrix only). Finite and infinite graphs, Definition of directed graphs, illustrative examples, Directed graphs, Applications of graphs. [sections 1.1, 1.2, 1.3, 1.4, 7.1, 9.1, 9.2]

Degree of a vertex, odd vertex, even vertex, relation between sum of degrees of vertices and the number of edges in a graph, and its consequence: number of odd vertices in a graph is even. Isolated vertex, pendant vertex, null graph, complete graphs [page 32], bipartite graphs [page 168], complete bipartite graph [page 192-prob 8.5], regular graph, complement* of a graph, graph isomorphisms, self complementary* graphs, illustrative examples. [sections 1.4, 1.5, 2.1]

Sub-graphs, edge disjoint sub-graphs, spanning sub-graphs*, induced subgraphs [sections 2.2]

The decanting problem and its graph model [no solution at this point]. The puzzle with multicolour cubes [problem 1.8 and section 2.3].

Module 2 Walks, open walks, closed walks, paths, circuits, end vertices of a path, path joinig two vertices, length of a path, connected and disconnected graphs. Components of a graph. [sections 2.4, 2.5]

Euler line, Euler graph, unicursal line, unicursal graph, characterisaion of Euler graph, Concept of Euler digraph [section 2.5, 9.5], Solution of the decanting problem. The Königsberg problem, the Chinese postman problem* and the Teleprinter's problem, their graph models and solutions. [problem 1.8 and sections 2.3, 1.2, 9.5]

Module 3 Trees- properties of trees, distance, eccentricity, center, radius, diameter, spanning tree, illustrative examples. [sections 3.1, 3.2, 3.3, 3.4, 3.7]

Planar graphs examples of planar and non-planar graphs, different representations of a planar graph. Regular polyhedra, Euler's polyhedral formula. [Theorem 5.6, without proof] . Illustrative examples, Kuratowski's graphs and their importance in

the theory of planar graphs, forbidden sub-graph, characterisaion of planar graph [Kuratowski's theorem, Theorem 5.9, without proof], illustrative examples-both planar and non-planar. [sections 5.2, 5.3, 5.4, 5.5] Graph theoretic version of the Four Colour Theorem, without proof.

TEXT: Narsingh Deo: Graph Theory with applications for Engineering and Computer Science, Prentice Hall of India Pvt. Ltd., 2000.

References:

1. BalakrishnanR and Ranganatahan: *A Text Book of Graph Theory*, Springer
2. Body J A and Murthy U S R:*Graph Theory with Applications*, The Macmillan Press
3. Harary F: *Graph Theory*, Addison-Wesley
4. Vasudev C: *Graph Theory with Applications*
5. West D B: *Introduction to Graph Theory*, Prentice Hall of India Pvt. Ltd.

Note: Generally, the references are from NARSINGH DEO. Those marked with an asterisk are found elsewhere.

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 18 hours; Module 2: 18 hours; Module 3: 18 hours