Syllabus for the First Degree Programme in Mathematics of the University of Kerala

Semester VI: Vector Analysis

CODE: MM 1643 — Instructional hours per week: 3; No. of credits: 3

OVERVIEW: The students are already aware of the concepts of a vector in three dimensional space and their dot, cross and triple products, as well as of derivatives of vector-valued functions, from the earlier semester. This course deals with advanced topics in vector calculus and covers Chapter 16 of the text. The authors claim that this chapter takes the student back to the roots of Calculus. The main theme is the concept of a flow and we analyse mathematically various types of flows.

Module 1 Vector fields, graphical representation, inverse square field, gradient fields, conservative fields and potential functions, divergence and curl, the ∇ operator, the Laplacian, ∇^2.

Line integrals, evaluation of line integrals, line integrals in 3-space, mass of a wire as a line integral, arc length as a line integral, line integral with respect x, y and$z$, line integral along piecewise smooth curves, change of parameter in a line integral, reversing the direction of integration, work as a line integral, work done by a vector field, work expressed in scalar form.

Module 2 Work integrals, independence of path, the fundamental theorem of work integrals, work integrals along closed paths, a test for conservative vector fields, conservative vector fields in 3-space, conservation of energy.

Green's theorem, finding work using Green's theorem, finding areas using Green's theorem, Green's theorem for multi-connected regions.

Surface integrals: definition, evaluation of surface integrals over $z = g(x, y)$, $y = g(x, z)$ and $x = g(y, z)$, mass of curved lamina as a surface integral, surface area as a surface integral.

Module 3 Application of surface integrals: flow fields, oriented surfaces, orientation of a smooth parametric surface, flux, evaluation of flux integrals, orientation of non-parametric surfaces.

Orientation of piecewise smooth closed surfaces, the Divergence theorem, using the Divergence theorem to find flux, divergence viewed as flux density, sources and sinks, Gauss' law for inverse square fields, Gauss' law in electrostatics.

Relative orientation of curves and surfaces, Stokes' theorem, using Stokes' theorem to calculate work, relationship between Green's theorem and Stokes' theorem, curl viewed as circulation.

TEXT: Howard Anton, et al: Calculus, Seventh Edition, John Wiley

References:

1. Thomas and Finney, *Calculus and Analytic Geometry*, Ninth Edition, Addison-Wesley.
2. Kreyzig, *Advanced Engineering Mathematics*, 8^{th} edition, John Wiley.
3. Peter V. O' Neil, *Advanced Engineering Mathematics*, Thompson Publications, 2007
4. Michael D. Greenberg, *Advanced Engineering Mathematics*, Pearson Education, 2002.

Distribution of instructional hours:

Module 1: 18 hours; Module 2: 18 hours; Module 3: 18 hours