Syllabus for the First Degree Programme in Mathematics of the University of Kerala

Semester V
Complex Analysis

CODE: MM 1542 — Instructional hours per week: 4
No.of credits: 3

Module 1 Review of real functions like $\sin x$, $\cos x$, e^x and their series representations. In order to introduce complex numbers, we first consider complex polynomials and rational functions. Then power series and convergence of power series are discussed. We then use power series to obtain some complex functions. (For example, the power series $1 + z + z^2 + \dots$ gives the complex function $\frac{1}{1-z}$ as its sum when $|z| < 1$.)

Similarly, we define e^z, $\sin z$, $\cos z$, etc. as the sums of certain power series. Next, we introduce the concepts of limit, continuity and differentiability through examples and counter-examples. Cauchy-Riemann equations are derived in Cartesian and polar coordinates.

Module 2 Properties of differentiable complex functions on open sets (called anlytic functions): for f and g analytic on an open set Ω, we have $f \pm g$, fg, f/g, where $g \neq 0$ are analytic on Ω. Taylor Series and Laurent Series (without proof) with illustrations.

Defintion of harmonic functions. Connection between harmonic and analytic functions. Harmonic conjugates. Determination of harmonic conjugates. Orthogonal family of curves. Connection between analytic functions and orthogonl family of curves.

Module 3 Mapping properties of $w = z^2$, $w = \frac{1}{z}$, $w = \sin z$, $w = e^z$. The concept of conformal mapping is introduced through various examples. Criterion for the conformality of the mapping $w = f(z)$ (without proof).

The concept of the Riemann sphere is introduced. Bilinear transformations. Properties of bilinear transformations. Decomposition of a bilinear transformation into special transformations such as translation, dialation and inversion. Special bilinear fractional transformations.

TEXT: R V Churchill and Brown: Functions of a Complex Variable

References:

1. J M Howie: *Complex Analysis*, Springer

2. V Karunakaran: *Complex Analysis*

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 24 hours; Module 2: 24 hours; Module 3: 24 hours