Syllabus for the First Degree Programme in Mathematics of the University of Kerala

Semester IV
Methods of Algebra and Calculus-II

CODE: MM 1441

Instructional hours per week: 5
No.of credits: 4

Overview of the course:

The first two modules on Algebra deal with polynomials, based on part of Chapter 14 and Chapter 15 and part of Chapter 16 of Childs's text. Our aim in the first module is to prove that any polynomial of degree greater than or equal to 1 with coefficients in a field factors uniquely (in a certain sense) into a product of irreducible polynomials. We use the division theorem, Euclid's algorithm and Bezout's identity in the process. This leads to the fundamental theorem of arithmetic.

In the first module on Calculus, some of the basic concepts of Calculus are extended to functions of two or more variables. Limits, continuity and derivatives of functions of two or more variables are defined. These derivatives are put to use in solving maximization and minimization problems and in the application of Lagrange multipliers.

In the second module, the concept of a definite integral is extended to functions of two and three variables. These are then put to use to calculate surface areas and volumes of solids.

Module 1: Methods of Algebra I

Before discussing this material, the notion of polynomials and of polynomial equations, studied in the Higher Secondary classes must be reviewed. The discussion on quadratic equations and their general solutions leads naturally to the question of higher degree polynomial equations. Section E of Chapter 16 of the text, on the solutions of the cubic and the fourth degree equations, can be discussed at this point. (See also
`http://www-history.mcs.st-andrews.ac.uk/HistTopics/Quadratic_etc_equations.html`)
The unsolvability of higher degree polynomials by radicals must be emphasized. Newton's method of finding approximate solutions of equations, as discussed in Section 7 of Chapter 4 of the Calculus text (Anton et al) can be discussed here.

Module 2: Methods of Algebra II

A discussion of polynomial over abstract rings as in Chapter 14 is to be done here. The distinction between polynomial as an algebraic expression and polynomial as function should be emphasized, as in the text. All the four sections of Chapter 15 are to be discussed next. We begin with the Division theorem in Section A, followed by the Remainder theorem and the Root theorem. Section B deals with primitive roots. Euclid's algoritm and Bezout's identity comprise Section D. Factorization into irreducible polynomials is discussed in Section D. The first three sections (A, B and C) of Chapter 16 lead to a discussion of Irreducible polynomials

over $\mathbb{R}$. The Fundamental Theorem of Algebra is to be stated, but need not be proved at this stage.

TEXT: Lindsay N. Childs, *A Concrete Introduction to Higher Algebra*. Second Edition, Springer

REFERENCES:

1. J B Fraleigh, *A First Course in Abstract Algebra*, Narosa Publications
2. I N Herstein, *Topics in Algebra*, Vikas Publications
3. J A Gallian, *Contemporary Abstract Algebra*, Narosa Publications
4. D A R Wallace, *Groups, Rings and Fields*, Springer
5. Jones and Jones, *Number Theory*, Springer

Module 3: Methods of Calculus-I

In this part of the course, we consider the calculus of functions of two variables. This serves as an introduction to *calculus of several variables*. It is based on Chapter 14 and Chapter 15 of the text.

After a discussion of functions of two variable and their graphs, as in the first section of Chapter 14, we discuss the concepts of limit and continuity of such functions. We then move on to a discussion of differentiation of functions of two variables, as in Sections 1–3, Section 5 and Sections 8–9 of Chapter 14. The section on differentiability and differentials (Section 14.5) and the sections on vectors (Section 14.6 and Section 14.7) need not be discussed.

Module 4: Methods of Calculus-II

Integration in space is to be done as in Sections 1–5 of Chapter 15. The last three sections of Chapter 15 need not be discussed. Section 15.1 deals with the definition, properties and mode of evaluating double integrals. Double integrals over non-rectangular regions are considered in Section 15.2. Reversal of the order of integration and calculation of area as a double integral are also discussed. Section 15.3 introduces double integrals in polar coordinates. Conversion of double integrals from rectangular to polar coordinates is also considered.

Parametric representation of surfaces is discussed in Section 15.4. A discussion of partial derivatives of vector-valued functions leads to tangent planes and parametric surfaces. Section 15.5 defines a triple integral, properties of triple integrals, and evaluation of triple integrals over rectangular boxes and other regions. We also compute volume using triple integrals and consider changing the order of integration.

TEXT: Howard Anton, et al, *Calculus*, Seventh Edition, John Wiley

REFERENCES:

1. James Stewart, *Essential Calculus*, Thompson Publications, 2007.
2. Thomas and Finney, *Calculus and Analytic Geometry*, Ninth Edition, Addison-Wesley.
3. S.Lang, *A First Calculus*, Springer.

Distribution of instructional hours:

Module 1: 20 hours; Module 2: 20 hours; Module 3: 25 hours, Module 4: 25 hours