University of Kerala
Complementary Course in Mathematics
for First Degree Programme in Physics

Semester IV
Mathematics-IV
(Complex Analysis, Fourier Series and Fourier Transforms)

CODE: MM 1431.1 — Instructional hours per week: 5
No. of Credits: 4

Module 1: Complex Analysis-I

1. *Complex Functions*

   - (Review only) Basic concepts about complex numbers. Real and imaginary parts, modulus of a complex number. Algebra of complex numbers, complex plane, modulus and argument of a complex number, n-th roots of a complex number.
   - Sets of points in the complex plane, circle, open disc, cloded disc, closed set open set, connected set and limit point of a set.
   - Complex functions. The real and imaginary parts of a complex function. Functions as mapping between two complex planes. Polynomial and rational functions. Definition of elementary functions-exp(z),sin$z$,cos$z$ etc by defining their real and imaginary parts in terms of known real functions. Definition of log$z$ as the inverse of exponential function and its multivalued nature. Principal branch of logarithm. Rational and complex powers of a complex number and their multi-valuedness.

2. *Complex differentiation*

   - The limit of a complex function. Limit in terms of real and imaginary parts of the function. Basic properties of limits. Derivative of a complex function. The Cauchy-Riemann equations and the necessary and sufficient conditions for differentiability. Analytic functions. Analyticity of the elementary functions.
   - Harmonic functions of two variables. The result that the real and imaginary parts of an analytic function are harmonic. Method of constructing an analytic function with a given harmonic function as real or imaginary part.

Module 2: Complex Analysis-II

1. *Complex Integration*

   - Curves in the complex plane. Smooth and piecewise smooth curves. Integral of a complex function along acurve. Evaluation of line integrals by reducing to definite integral.
   - Cauchy's theorem (without proof) and its implications. Conditions for independence of path in simply connected domains. Fundamental theorem showing connection between line integral of a function and its and anti-derivative (without proof). Cauchy's integral formula for derivatives and its use in computing line integrals over simple closed curves.

2. *Complex series*

    - Sequences and series of complex numbers and their convergence. Cauchy's convergence principle Comparison and ratio tests for convergence of compex series. Power series and radius of convergence of a power series.
    - Taylor series and Taylor's theorem on the representation of a function analytic in an open disk and the uniqueness of such representation (without proof). Taylor series representation of elementary functions.
    - Laurent's theorem on the representation of a function analytic in an annulus and the uniqueness of such representation (without proof). Examples of finding series representations of simple functions.

3. *Residue Theory*

    - Isolated singular point of a complex function and classification of such singularities-removable singularity, poles and essential singularity. Residue of a function at a singular point. Calculation of residues. Cauchy's residue theorem(without proof). Evaluation of line integrals using Residue theorem. Use of Residue theorem in evaluating definite integrals of rational functions involving sines and cosines.

Module 3: Fourier Series and transforms

- Periodic functions, trigonometric series, Fourier series, evaluation of Fourier coefficients for functions defined in $(-\infty, +\infty)$, Fourier series for odd and even functions, half range series, Fourier series for odd and even functions, Fourier series of functions defined in $(-L, +L)$.
- Fourier integrals and Fourier transforms.

Text for Modules 1 and 2: Ruel V. Churchill and James Ward Brown,*Complex Variables and Applications*, 1989.

Text for Module 3: Kreyzig, *Advanced Engineering Mathematics*, 8^{th} edition, John Wiley. Chapter 8, Sections 1, 2, 3, 4, 8, 10.

References

1. *Advanced Engineering Mathematics*, Peter V. O' Neil, Thompson Publications, 2007
2. *Advanced Engineering Mathematics*, Michael D. Greenberg, Pearson Education, 2002.