Syllabus for the First Degree Programme in Mathematics
of the University of Kerala

Semester III
Methods of Algebra and Calculus-I

CODE: MM 1341 — Instructional hours per week: 5
No.of credits: 4

Overview of the course:

Continuing the discussion on number theory in the first two semesters, here we make first contact with the part of mathematics currently called *Abstract Algebra*. It is based on Chapter 8, Chapter 9, Chapter 11 and Chapter 12 of the same text. As the author points out, the idea is to help students appreciate the basic concepts of abstract algebra, with the grounding they have in the algebraic theory of integers and polynomials. Contrary to the usual stand-alone courses on abstract algebra, we introduce *rings* before *groups*, since the former arise naturally as generalizations of number systems. The algebraic properties of the ring of integers is developed. Groups are introduced subsequently. The concepts of ring homomorphism and group homomorphism are also discussed in detail. The material covered is based on Chapters 8, 9, 11 and 12 of the same text.

The geometry and calculus done in the first two semesters were confined to two dimensions. Here we consider surfaces and curves in three dimensions. It is based on Chapters 12 and 13 of the text. Chapter 12 discusses analytical geometry of lines, planes and other basic surfaces. The study of *vectors* is also done alongside. Later on, *cylindrical* and *spherical coordinates* are also introduced.

Parametric curves in 2-space and 3-space are best studied by *vector-valued functions*. In Chapter 13, we discuss the Calculus of vector-valued functions in detail, discussing limits, derivatives and integrals of vector-valued functions. We also study their applications to Physics, especially to the derivation of *Kepler's Laws of Planetary Motion*.

Module 1: Methods of Algebra I

Chapter 8 introduces the concept of *rings* and *fields*. We discuss $\mathbb{Z}/m\mathbb{Z}$ and the particular case of $\mathbb{Z}/p\mathbb{Z}$, p prime, being a field. The idea of a *ring homomorphism*, along with its basic properties, is introduced. The chapter ends with a proposition on a *ring isomorphism*. The problems are to be given due importance; while some problems help fix the concepts, others add to the ideas already covered in the text.

Chapter 9 begins with a definition of orders of elements, followed by *Fermat's* and *Euler's theorems*. The method of finding high powers modulo m is also to be discussed. Next comes a discussion on the units of the ring of congruence classes leading to the definition of an abstract *group* and then the *group of units* of an abstract ring, as in Section E and Section F of Chapter 9. This culminates in the *Abstract Fermat's Theorem*, as in Section 9E. The exponent of an Abelian group, as in Section 9F also has to be discussed.

Module 2: Methods of Algebra II

Further development of the theory of groups is seen in Chapter 11. It begins with the defintion of a *subgroup*, followed by *Lagrange's theorem*. Euler's theorem and Fermat's theorem are given as consequences of Lagrange's theorem. *A probabilistic primality test* as an application of Lagrange's theorem is to be discussed as in Section 11C. Section 11D on *homomorphisms* and the case of an *isomorphism* between rings leading to an isomorphism between the corresponding groups of units are to be discussed in detail. Some examples on *non-abelian groups* are found in Section E and the Chapter ends with *Cayley's theorem*.

As an illustration of the interplay between number theory and abstract algebra, we consider the *The Chinese Remainder Theorem* and as an application, reprove the multiplicative property of the phi function done earlier (Corollary 3 of Section 12C). The square roots of 1 modulo some integer, as in Section C of Chapter 12 must also be discussed.

TEXT: Lindsay N. Childs, *A Concrete Introduction to Higher Algebra*. Second Edition, Springer

REFERENCES:

1. J B Fraleigh, *A First Course in Abstract Algebra*, Narosa Publications
2. I N Herstein, *Topics in Algebra*, Vikas Publications
3. J A Gallian, *Contemporary Abstract Algebra*, Narosa Publications
4. D A R Wallace, *Groups, Rings and Fields*, Springer
5. Jones and Jones, *Number Theory*, Springer

Module 3: Vectors in Three Dimensional Space

Students have had some exposure to analytic geometry in three dimensions (such as the equations to planes and lines) and to vectors, in their Higher Secondary Classes. These must be reviewed with more illustrations. Here the aid of a plotting software becomes essential. The Free Software GNUPLOT, mentioned earlier, has 3D capabilities also. All sections of Chapter 12 must be discussed.
(see also `http://mathworld.wolfram.com/topics/Surfaces.html`)

Chapter 12 introduces 3-dimensional spaces. Here we learn to derive the equations of spheres and cylindrical surfaces and sketch their graphs. Vectors are introduced in Sec 12.2. The dot product and its application to work done by a force are also discussed. This is followed by a study of the cross product and its geometric interpretation. This leads us to the scalar triple product.

Sec 12.5 discusses the parametric representation of a line and Sec 12.6 is on planes in 3-space. Quadric surfaces, which are three dimensional analogues of conic sections, are covered in Sec 12.7, while in Sec 12.8, two new types of coordinates—cylindrical and spherical coordinates, are introduced.

Module 4: Vector-valued Functions

The next topic of discussion is the calculus of *vector valued functions* and its application to geometry as in Chapter 13. All sections of this Chapter must be discussed.

Sec 13.1 introduces vector-valued functions, which find a lot of applications in Physics and engineering. We learn to sketch the graphs of vector-valued functions and write down the vector form of a line segment. Sec 13.2 defines limits, derivatives and integrals of vector-valued functions, as well as their properties. Sec 13.3 begins with *smooth parametrization*. We learn to look at arc length from the vector view-point. *Arc length parametrization* is also introduced. This is followed by a detailed discussion of change of parameter and arc length parametrization.

In Sec 13.4, we discuss *unit tangent vectors* and *unit normal vectors* and compute **T** and **N** for curves paramterized by arc lengths. *Binormal vectors* in 3-space are introduced towards the end of the section. Sec 13.5 deals with *curvature* and derives formulas for curvature and *radius of curvature*. Exercises 17 (a) and 17 (b) of Exercise Set 13.5 on curvature of plane curves and some of its applications in the subsequent exercises must be discussed in detail.

The motion of a prticle along a curve is discussed in Sec 13. Its velocity, acceleration and speed are computed. The *normal* and *tangent components* of acceleration are studied, as also the *motion of a projectile*. In Sec 13.7, *Kepler's laws of planetary motion* are discussed in detail, using the notion of *central forces*.

TEXT: Howard Anton, et al, *Calculus*. Seventh Edition, John Wiley

REFERENCES:

1. James Stewart, *Essential Calculus*, Thompson Publications, 2007.
2. Thomas and Finney, *Calculus and Analytic Geometry*, Ninth Edition, Addison-Wesley.
3. S.Lang, *A First Calculus*, Springer.

Distribution of instructional hours:

Module 1: 25 hours; Module 2: 25 hours; Module 3: 20 hours, Module 4: 20 hours