

University of Kerala
Complementary Course in Mathematics
for First Degree Programme in Physics

Semester II
Mathematics-II
(Integration and Vectors)

CODE: MM 1231.1

Instructional hours per week: 4
No. of Credits: 3

Overview of the course:

The complementary course in the second semester continues the trend indicated in the first, namely, laying emphasis on applications of integral calculus and vectors to problems in Physics. Module 1 consists of a review of basic integration techniques and the applications of integration. It also covers multiple integrals. Modules 2 and 3 deal with vector calculus and its applications in detail.

Module 1: Integration (with applications to Physics)

- Indefinite integrals (Review only), integral curves, integration from the view point of differential equations, direction fields
Exercise set 5.2; Questions 43, 44 and 51.
- (Review only) Definite integral and Fundamental Theorem of Calculus.
- Rectilinear motion: finding position and velocity by integration. Uniformly accelerated motion. The free-fall model. Integrating rates of change. Displacement in rectilinear motion. Distance travelled in rectilinear motion. Analysing the velocity versus time curve. Average value of a continuous function. Average velocity revisited.
Exercise set 5.7; Questions 3, 4, 5, 6, 29, 39, 45 and 55.
- Review of integration techniques.
- Use of definite integrals in finding area under curves, area between two curves, volume of revolution, arc length and surface area of a solid of revolution.
- The idea of approximating the volume under a bounded surface in 3-space by volumes of boxes, leading to the definition of double integrals of functions of two variables over bounded regions. Evaluation of double integrals by iterated integrals. Evaluation by changing to polar co-ordinates and by suitably changing order of integration in the iterated integral. Applications to finding the volume of solids under bounded surfaces.
- Triple integrals over bounded regions in three space. Evaluation by iterated integrals. Cylindrical coordinates and spherical coordinates and their relation to Cartesian coordinates. Use of cylindrical and spherical co-ordinates in evaluating triple integrals. Applications of triple integrals to finding volumes of solid objects.

Module 2: Vector Differentiation

- (Review only) Vectors in 3-space. Addition of two vectors, multiplication of a vector by a scalar and basic properties of these operations. Representation in Cartesian coordinates using standard basis. Dot, cross and triple product of vectors, their significance and properties.
- Vector function of a single variable and representation in terms of standard basis. Limit of a vector function and evaluation of limit in Cartesian representation. Continuous vector functions and the idea that such functions represent oriented space curves. Examples.
- Derivative of a vector function and its geometric significance. Derivative in terms of Cartesian components. Tangent vector to a curve, smooth and piecewise smooth curves. Applications to finding the length and curvature of space curves, velocity and acceleration of motion along a curve etc.
- Scalar field and level surfaces. The gradient vector of a scalar field (Cartesian form) at a point and its geometric significance. Gradient as an operator and its properties. Directional derivative of a scalar field and its significance. Use of gradient vector in computing directional derivative.
- Vector fields and their Cartesian representation. Sketching of simple vector fields in the plane. The curl and divergence of a vector field (Cartesian form) and their physical significance. The curl and divergence as operators, their properties. Irrotational and solenoidal vector fields. Various combinations of gradient, curl and divergence operators.

Module 3: Vector Integration

- The method of computing the work done by a force field in moving a particle along a curve leading to the definition of line integral of a vector field along a smooth curve. Scalar representation of line integral. Evaluation as a definite integral. Properties. Line integral over piecewise smooth curves. Green's theorem in the plane (without proof) for a region bounded by a simple closed piecewise smooth curve.
- Oriented surfaces. The idea of flux of a vector field over a surface in 3-space. The surface integral of a vector field over a bounded oriented surface. Evaluation by reducing to a double integral. Use of cylindrical and spherical co-ordinates in computing surface integral over cylindrical and spherical surfaces.
- Stokes' theorem (without proof) for an open surface with boundary a piecewise smooth closed curve. Gauss' divergence theorem (without proof). Verification of the theorems in simple cases and their use in computing line integrals or surface integrals which are difficult to evaluate directly. Physical interpretation of divergence and curl in terms of the velocity field of a fluid flow.
- Conservative fields and potential functions. Relation of conservative vector fields to their irrotational nature and the path-independence of line integrals in the field (without proof). Significance of these results in the case of conservative force fields such as gravitational, magnetic and electric fields. Method of finding the potential function of a conservative field.

TEXT: Howard Anton, et al, *Calculus*. Seventh Edition, John Wiley

REFERENCES:

1. James Stewart, *Essential Calculus*, Thompson Publications, 2007.
2. Thomas and Finney, *Calculus and Analytic Geometry*, Ninth Edition, Addison-Wesley.
3. Peter V. O' Neil, *Advanced Engineering Mathematics*, Thompson Publications, 2007
4. Michael D. Greenberg, *Advanced Engineering Mathematics*, Pearson Education, 2002.

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 24 hours; Module 2: 24 hours; Module 3: 24 hours