Syllabus for the First Degree Programme in Mathematics of the University of Kerala

Semester II
Foundations of Mathematics

CODE: MM 1221 — Instructional hours per week: 4
No.of credits: 3

Overview of the course:

The present course is meant to be a foundation course in Mathematics. Here we examine some of the basic concepts on which the edifice of mathematics is built. As a strong structure naturally demands a strong foundation, the concepts put down here should be given due emphasis. The first module lays the prerequisites of algebra and the second that of Calculus.

Module 1: Foundations of Algebra

Though the students have learnt the *Principle of Mathematical Induction* in their Higher Secondary class, they may not have recognized it as providing *a canonical method of proof* in proceeding from one natural number to the next. This should be emphasized through examples and exercises. The logical equivalence of the two forms of induction and the contexts where one is more useful than the other are to be explained and illustrated with examples, as in the text. The application of these principles in other branches of mathematics must also be noted, as in Example 5 of Section A and Exercise 4 of Section B.

Next comes the *Well-Ordering Principle*. This is to be introduced as another form of the principle of induction, as in Section C of the text and its equivalence with the first principle of induction is to be proved as in Theorem 2 and Exercise E2.

Congruence modulo m, done in semester I is now presented as an equivalence relation and the congruence classes (mod m) are discussed through examples such as $\mathbb{Z}/2\,\mathbb{Z}$, and $\mathbb{Z}/2\,\mathbb{Z}$ (clock arithmetic), leading to the general case of $\mathbb{Z}/m\,\mathbb{Z}$.

Equipped with these concepts as foundations we continue with the theory of numbers, discussing arithmetic modulo m and a complete set of representatives in $\mathbb{Z}/m\,\mathbb{Z}$. Later, we discuss units, withe special reference to units in $\mathbb{Z}/m\,\mathbb{Z}$. As applications, only Section A on round robin tournaments and Section C on trial division need be discussed.

Next we move on to Fermat's and Euler's Theorems, as in Chapter 9. Only the first four sections of this chapter need be done. (The other sections are postponed to the next semester.). In Section C, exercises E7–E10 on the computation of Euler's phi function must be done and used to compute the phi-value of some specific numbers. As an application of the material in this chapter, Section B of Chapter 10 on RSA codes is to be discussed. (See `http://en.wikipedia.org/wiki/RSA`)

TEXT: Lindsay N. Childs, *A Concrete Introduction to Higher Algebra*. Second Edition, Springer

Module 2: Foundations of Calculus and Analytic Geometry

The concept of the limit of a function was discussed in the first semsester, with sufficient examples, but without going into too much rigour. A rigorous tretment of the limit of a function will be done in this semester, as in section 2.4 of Chapter 2 of the text. The motivation for the definition of a limit and the transition from the informal to the formal $\epsilon - \delta$ definition should be given due emphasis. Example 2 is an illustartion of a general form of a limit proof. The fact that the value of δ is not unique should be brought out. Limits as $x \to \infty$, as well as infinite limits are also to be discussed more rigorously.

The remaining part of the course continues the corresponding part of the Semester I course. It is based on Chapters 5-8 of the same text.

We start with a discussion of inverses of functions, as in Chapter 7. Though the students may have used this idea in the Higher Secondary class, this has to be done in a more thorough manner. Also, the ideas have to be graphically interpreted. Before discussing the exponential and logarithmic functions, the idea of irrational exponents has to be made clear, as in Section 7.2. After this, the definition and basic properties of inverse trigonometric functions can be done, as in Section 7.6. Then the derivatives and integrals of all these can be discussed as in Sections 7.3, 7.4 and 7.6. (Section 7.5 on logarithmic functions approached via integrals need not be discussed). The discussion on hyperbolic functions, as in Section 7.8 can be done next. (See also `http://en.wikipedia.org/wiki/Catenary`) Then L'Hospital's rule, as in Section 7.7 after this.

Some applications of integration, as in Chapter 5 and Chapter 6 are to be discussed next, followed by various techniques of integration, as in Chapter 8, excluding the last three sections on Table of integrals (Section 8.6), Numerical Integration (Section 8.7) and improper integrals (Section 8.8).

Polar coordinates in co-ordinate geometry and expressions for slope, area, arc length and so on for curves given by polar coordinates are to be discussed as in Sections 11.1–11.3. The polar equations of conics, as in Section 11.6 are also to be discussed.

TEXT: Howard Anton, et al, *Calculus*. Seventh Edition, John Wiley

REFERENCES:

1. James Stewart, *Essential Calculus*, Thompson Publications, 2007.

2. Thomas and Finney, *Calculus and Analytic Geometry*, Ninth Edition, Addison-Wesley.

3. S.Lang, *A First Calculus*, Springer.