University of Kerala
Complementary Course in Mathematics
for First Degree Programme in Chemistry

Semester I
Mathematics-I
(Differentiation and Matrices)

CODE: MM 1131.2 — Instructional hours per week: 4
No. of Credits: 3

Overview of the course:

The complementary course intended for Chemistry students lays emphsis on the application of mathematical methods to Chemistry. The two modules on Calculus links the topic to *the real world and the student's own experience* as the authors of the text put it. Doing as many of the indicated exercises from the text should prove valuable in understanding the applications of the theory. Applications to Chemistry on the lines of those in Physics as given in the text could be obtained from the net. The third module covers matrix theory.

Module 1: Differentiation with applications to Chemistry-I

- Functions and graphs of functions with examples from Chemistry. Interpretations of slope. The graph showing direct and inverse proportional variation. Mathematical models (functions as models). Parametric equations. Cycloid.

  Exercise set 1.8; Questions $31 - 34$.

- Instantaneous velocity and the slope of a curve. Limits. Infinite limits and vertical asymptotes. Limits at infinity and horizontal asymptotes. Some basic limits. Indeterminate forms of the type $0/0$.

  Exercise set 2.1; Questions 27 and 28.

- Continuity. Slopes and rates of change. Rates of change in applications. Derivative.

  Exercise set 3.1; Questions 1, 2 and 16.

- Techniques of differentiation. Higher derivatives. Implicit differentiation. Related rates. Local linear approximation. Differentials. Examples $1 - 6$.

  Exercise set 3.8; Questions $53 - 55$.

- Rectilinear motion. Speeding up and slowing down. Analysing the position versus time curve. Free fall motion.

  Examples $1 - 7$. Exercise set 4.4; Questions 8, 9, $30 - 32$.

- Absolute maxima and minima. Applied maximum anmd minimimum problems.

  Exercise set 4.6; Questions 47 and 48.

- Statement of Rolle's Theorem and Mean Value Theorem. The velocity interpretation of Mean Value Theorem. Statement of theorems 4.1.2 and 4.83 (consequences of the Mean Value Theorem).

- Inverse functions. Continuity and differentiability of inverse functions. Graphing inverse functions. exponential and logarithmic functions. Derivatives of logarithmic functions and logarithmic differentiation. Derivatives of the exponential function. Graphs and applications involving logarithmic and exponential functions.

  Exercise set 7.4; Question 50.

- L'Hospital's Rule for finding the limits (without proof) of indeterminate forms of the type $0/0$ and ∞/∞. Analysing the growth of exponential functions using L'Hospital's Rule. Indeterminate forms of type $0 \cdot \infty$ and $\infty - \infty$ and their evaluation by converting them to $0/0$ or ∞/∞ types. Indeterminate forms of type 0^0, ∞^0 and 1^∞.

- Definitions of hyperbolic functions. Graphs of hyperbolic functions. Hyperbolic identities. Why they are called hyperbolic functions. Derivatives of hyperbolic functions. Inverse hyperbolic functions. Logarithmic forms of inverse hyperbolic functions. Derivatives of inverse hyperbolic functions.

Module 2: Differentiation with applications to Chemistry-II

- Power series and their convergence. Results about the region of convergence of a power series(without proof). Radius of convergence. Functions defined by a power series. Results about term by term differentiation and integration of power series (without proof). Taylor's theorem with derivative form of remainder (without proof) and its use in approximating functions by polynomials. Taylor series and Maclaurin's series and representation of functions by Taylor series. Taylor series of basic functions and the regions where these series converge to the respective functions. Binomial series as a Taylor series and its convergence. Obtaining Taylor series representation of other functions by differentiaion, integration, substitution etc.

- Functions of two variables. Graphs of functions of two variables. Equations of surfaces such as sphere, cylinder, cone, paraboloid, ellipsoid, hyperboloid etc. Partial derivatives and chain rule (various forms). Euler's theorem for homogeneous functions. Jacobians.

  Exercise set 14.3; Questions 47 and 48.

  Exercise set 14.4; Question 50.

  Exercise set 14.5; Question 42.

- Local maxima and minima of functions of two variables. Use of partial derivatives in locating local maxima and minima. Lagrange method for finding maximum/minimum values of functions subject to one constraint.

  Exercise set 14.9; Question 20.

Module 3: Theory of Matrices

- (Review only) basic concepts about matrices. Operations involving matrices, different types of matrices. Representation of a system of linear equation in matrix form. Inverse of a matrix, Cramer's rule.

- The rows and columns of a matrix as elements of $\mathbb{R}^n$ for suitable n. Rank of a matrix as the maximum number of linearly independent rows/columns. Elementary row operations. Invariance of rank under elementary row operations. The Echelon form and its uniqueness. Finding the rank of a matrix by reducing it to echelon form.

- Homogeneous and non-homogeneous system of linear equations. Results about the existence and nature of solution of a system of equations in terms of the ranks of the matrices involved.
- The eigen value problem. Method of finding the eigen values and eigen vectors of a matrix. Basic properties of eigen values and eigen vectors. Eigen values and eigen vectors of a symmetric matrix.
- Diagonalisable matrices. Advantages of diagonalisable matrices in computing matrix powers and solving system of equations. The result that a square matrix of order n is diagonalisabe (i) if and only if it has n linearly independent eigen vectors (ii) if it has n distinct eigen values. Method of diagonalising a matrix. Diagonalisation of real symmetric matrices. Similar matrices.

Text for Modules 1 and 2: Howard Anton, et al, *Calculus*. Seventh Edition, John Wiley

Text for Module 3: David C. Lay, *Linear Algebra*, Thompson Publications, 2007

References:

1. James Stewart, *Essential Calculus*, Thompson Publications, 2007.
2. Thomas and Finney, *Calculus and Analytic Geometry*, Ninth Edition, Addison-Wesley.
3. Peter V. O' Neil, *Advanced Engineering Mathematics*, Thompson Publications, 2007

Distribution of instructional hours:

Module 1: 24 hours; Module 2: 24 hours; Module 3: 24 hours