University of Kerala
Complementary Course in Mathematics
for First Degree Programme in Physics

Semester I
Mathematics-I
(Differentiation and Analytic Geometry)

CODE: MM 1131.1 — Instructional hours per week: 4
No. of Credits:3

Overview of the course:

The complementary course intended for Physics students lays emphsis on the application of mathematical methods to Physics. The two modules on Calculus links the topic to *the real world and the student's own experience* as the authors of the text put it. Doing as many of the indicated exercises from the text should prove valuable in understanding the applications of the theory. Analytic geometry presented here is important in applications of calculus.

Module 1: Differentiation with applications to Physics-I

- Functions and graphs of functions with examples from Physics. Interpretations of slope. The graph showing direct and inverse proportional variation. Mathematical models (functions as models). Parametric equations. Cycloid and Brachistochrone problem.

  Exercise set 1.8; Questions $31 - 34$, 37 and 39.

- Instantaneous velocity and the slope of a curve. Limits. Infinite limits and vertical asymptotes. Limits at infinity and horizontal asymptotes. Some basic limits. Indeterminate forms of the type $0/0$.

  Exercise set 2.1; Questions 27 and 28.

- Continuity. Slopes and rates of change. Rates of change in applications. Derivative.

  Exercise set 3.1; Questions $1 - 4$ and 15, 16, $18 - 21$.

  Exercise set 3.2; Question 39.

- Techniques of differentiation. Higher derivatives. Implicit differentiation. Related rates. Local linear approximation. Differentials. Examples $1 - 6$.

  Exercise set 3.3; Question 68.

  Exercise set 3.4; Question 32.

  Exercise set 3.8; Questions $57 - 60$.

- Rectilinear motion. Speeding up and slowing down. Analysing the position versus time curve. Free fall motion.

  Examples $1 - 7$. Exercise set 4.4; Questions 8, 9, 23, 27, $30 - 32$.

- Absolute maxima and minima. Applied maximum anmd minimimum problems.

  Exercise set 4.6; Questions 47, 48, 56, 59.

- Statement of Rolle's Theorem and Mean Value Theorem. The velocity interpretation of Mean Value Theorem. Statement of theorems $4.1.2$ and 4.83 (consequences of the Mean Value Theorem).

  Exercise set 4.8; Questions $22-25$.

- Inverse functions. Continuity and differentiability of inverse functions. Graphing inverse functions. exponential and logarithmic functions. Derivatives of logarithmic functions and logarithmic differentiation. Derivatives of the exponential function. Graphs and applications involving logarithmic and exponential functions. Logistic curves. Example 4 of section 7.4 (Newton's Law of Cooling).

  Exercise set 7.4; Questions 31, 35, $49-50$.

- L'Hospital's Rule for finding the limits (without proof) of indeterminate forms of the type $0/0$ and ∞/∞. Analysing the growth of exponential functions using L'Hospital's Rule. Indeterminate forms of type $0 \cdot \infty$ and $\infty - \infty$ and their evaluation by converting them to $0/0$ or ∞/∞ types. Indeterminate forms of type 0^0, ∞^0 and 1^∞.

  Exercise set 7.7; Questions 55.

- Definitions of hyperbolic functions. Graphs of hyperbolic functions. Hanging cables and other applications. Hyperbolic identities. Why they are called hyperbolic functions. Derivatives of hyperbolic functions. Inverse hyperbolic functions. Logarithmic forms of inverse hyperbolic functions. Derivatives of inverse hyperbolic functions.

  Exercise set 7.8; Questions 69 and 72.

Module 2: Differentiation with applications to Physics-II

- Power series and their convergence. Results about the region of convergence of a power series(without proof). Radius of convergence. Functions defined by a power series. Results about term by term differentiation and integration of power series (without proof). Taylor's theorem with derivative form of remainder (without proof) and its use in approximating functions by polynomials. Taylor series and Maclaurin series and representation of functions by Taylor series. Taylor series of basic functions and the regions where these series converge to the respective functions. Binomial series as a Taylor series and its convergence. Obtaining Taylor series representation of other functions by differentiaion, integration, substitution etc.

- Functions of several variables. Graphs of functions of two variables. Equations of surfaces such as sphere, cylinder, cone, paraboloid, ellipsoid, hyperboloid etc. Partial derivatives and differentials. The chain rule (various forms). Euler's theorem for homogeneous functions. Jacobians.

  Exercise set 14.3; Questions 47 and 48.

  Exercise set 14.4; Questions 49 and 50.

  Exercise set 14.5; Questions 41. 42 and 46.

- Local maxima and minima of functions of two variables. Use of partial derivatives in locating local maxima and minima. Lagrange method for finding maximum/minimum values of functions subject to one constraint.

  Exercise set 14.9; Question 20.

Module 3: Analytic Geometry

- Geometric definition of a conic-the focus, directrix and eccentricity of a conic. Classification of conics into ellise, parabola and hyperbola based on the value of eccentricity. Sketch of the graphs of conics. Reflection properties of conic sections.

  Exercise set 11.4; Questions $39 - 43$.

- Equations of the conics in standard positions. Equations of the conics which are translated from standard positions vertically or horizontally. Parametric representation of conics in standard form. Condition for a given straight line to be a tangent to a conic. Equation of the tangent and normal to a conic at a point.

- Asymptotes of a hyperbola. Equation of the asymptotes. Rectangular hyperbola and its parameric representation. Equation of tangent and normal to a rectangular hyperbola at a given point.

- Rotation of co-ordinate axes. Equation connecting the co-ordinates in the original and rotated axes. Elimination of the cross product term in a general second degree equation by suitable rotation. Identifying conics in non-standard positions represented by general second degree equation by suitable rotation of axes. The discriminant of a general second degree equation and its invariance under rotation of co-ordinate axes. The conditions on the discriminant for the general second degree equation to represent a conic, a pair of straight lines or a circle.

- Conic sections in polar coordinates. Eccentricity of an ellipse as a measure of flatness. Polar equations of conics. Sketching conics in polar coordinates. Kepler's Laws. Example 4 of section 11.6.

TEXT: Howard Anton, et al, *Calculus*. Seventh Edition, John Wiley

REFERENCES:

1. James Stewart, *Essential Calculus*, Thompson Publications, 2007.
2. Thomas and Finney, *Calculus and Analytic Geometry*, Ninth Edition, Addison-Wesley.
3. Peter V. O' Neil, *Advanced Engineering Mathematics*, Thompson Publications, 2007

DISTRIBUTION OF INSTRUCTIONAL HOURS:

Module 1: 24 hours; Module 2: 24 hours; Module 3: 24 hours